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METHODS OF ESTIMATING NON-RESPONSE OF MULTI-AUXILIARY INFORMATION WITH APPLICATION

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ABSTRACT

Sampling methods are often accompanied by sampling errors in collecting data. They have associated with the design the chosen sample which can be handled in some way or another based on theoretically known styles in this field or by using the comprehensive census type. However, the people concerned with preparing and implementing statistical work face non-random errors. Which are not less dangerous than errors connected with sampling method. Whether what has been chosen partially of the population or by containing all items. These non-random errors weaken the collected data efficiency. Because it is difficult to discover or to know: That is due to non-technical methods to handle them. In this paper focuses on the estimation of non-response of multi-auxiliary information of a finite population and infinite population. A comparison study is made between three methods of estimation using the multi-auxiliary information; these methods are multi-mean imputation, multi-ratio method of imputation and multi-power transformation method of imputation, through a randomized response technique. The relative efficiency was used to conclude the best methods by using empirically study (real data and simulation).

KEYWORDS: *Multi-Auxiliary Information, Multi-Ratio Estimator, Multi-Power Transformation, Empirical Study*

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1. INTRODUCTION

Sampling methods are often accompanied by sampling errors in collecting data. They have associated with the design the chosen sample which can be handled in some way or another based on theoretically known styles in this field or by using the comprehensive census type. However, the people concerned with preparing and implementing statistical work face non-random errors. Which are not less dangerous than errors connected with sampling methods. Data are subjected to non-random errors, whether collected from some items of the population or all the components of the population, meaning that, they do not decrease by increasing size of the sample as in the random errors. Missing data is a very common problem in most empirical research areas. The problem of missing data in survey sampling is called the problem of nonresponse. Missing data is present if the researcher fails to get the information from the sample. Different reasons can cause non-response such as the investigator refusal to answer, inaccessible, unable to answer, lack of information and so on. These non-random errors weaken the collected data efficiency because it is difficult to discover or to know. That is due to non-technical methods to handle them, where the non-response or missing data represents a huge problem in many studies and scientific researches Singh and Deo (2003). Undoubtedly, the sometimes of failure to account for the stochastic nature of missing data or nonresponse data can spoil the nature of data. Nature incomplete random data may be lead to distort the nature of original data. The auxiliary information has been used in improving the precision of the estimate of a parameter (see Cochran (1977)). Auxiliary variables are used to improve the efficiency of estimators at the estimation stage and it could be available in several forms. These errors are divided into complete non- responsiveness or partial non- responsiveness. The efficiency of a biased estimator is measured by the reciprocal of the amount of its mean square error (MSE). Thus the smaller the MSE the more the precision/efficiency of the estimator. In many sample surveys reduction in MSE, even by a very small amount, plays an important role and increases efficiency significantly of the over-all estimators. For more details on such methods, one can refer to Singh (2001), Singh and Horn (2000), Bratley. et al (2011), Aziz (2015) and Garcia and Cebrian (1996).

Assuming simple random sampling, we present three methods of estimation, Multi-Mean, Multi-Ratio and Multi-Power Transformation to estimate Non-Response of Multi-Auxiliary Information. In general, the power transformation estimator is shown to possess a smaller variance than the Mean and the Ratio estimators, see Almongy (2012). We compare between the results of these methods of estimation using empirical study.

2. MULTI-AUXILIARY INFORMATION

Let $\bar{Y} = N^{-1}\sum_{i=1}^{N} y_i$be the mean of the finite population$\Omega = \{1,2,\dots N\}$,a simple random sample without-replacement,s of size $n$is drawn from Ω to estimate $\bar{Y}$. Let r be the number of responding units out of sampled n units. Let the set of responding units be denoted by A and that of non-responding units be denoted by$A^{\backslash}$. For every unit $i \in A$, the value y_i is observed. However, for the units $i \in A^{\backslash}$, the y_i values are missing and imputed values are derived. We assume that imputation is carried out with the aid of multi-auxiliary variable,

$$X = (x_{ij})_{n\times p}, (i = 1,2,\dots,n;\ j = 1,2,\dots,p)$$

Such that x_{ij}, the value of x for unit i,and auxiliaryvariable j, is known and positive for every$i \in s = A \cup A^{\backslash}$. In other words, the data $X_{js} = \{x_{ij} : i \in s\}$are known. Following the notation of the Singh and Deo (2003). Singh and Deo (2003) present the case of single value imputation depending single auxiliary variable, but we present the case of single value imputation depending on multi-auxiliary variables. Let

$X = (x_{ij})_{n\times p}, (i = 1,2,\dots,n;\ j = 1,2,\dots,p)$be the $n \times p$ matrix ofthe$p-$auxiliary vectors associated with the study variable y,such that

$$X = \begin{pmatrix} x_{11} & \cdots & x_{1p} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{np} \end{pmatrix} = (x_{ij})_{n\times p}$$

It is assumed that full information is available on the multi-auxiliary variables, but responses are missing only for the study variable. In this situation, we suggest here some method of imputation following the notation of the Singh and Deo (2003).

2.1. Multi-Mean Imputation

Under the mean method of imputation, the data after imputation take the form

$$y_i^{\star} = \begin{cases} y_i \; if \; i \in A \\ \bar{y}_r \; if \; i \in A^{\backslash} \end{cases} \tag{1}$$

where

$$\bar{y}_{mm} = r^{-1} \sum_{i=1}^{r} y_i = \bar{y}_r, \tag{2}$$

$$\bar{y}_s = \frac{1}{n} \sum_{i=1}^{n} y_i^{\star}, \tag{3}$$

such that

$$\bar{y}_r = r^{-1} \sum_{i=1}^{r} y_i , \bar{x}_{rj} = r^{-1} \sum_{i=1}^{r} x_{ij} \; and \bar{x}_{sj} = n^{-1} \sum_{i=1}^{n} x_{ij}$$

2.2. Multi-Ratio Estimator

This method of imputation is called the multi-ratio method of imputation. Under this method of imputation, the data becomes

$$y_i^{\star} = \begin{cases} y_i, i \in A \\ \bar{y}_r \left[n \prod_{j=1}^{p} \left(\frac{\bar{x}_{nj}}{\bar{x}_{rj}} \right) - r \right] \frac{\sum_{j=1}^{p} x_{ij}}{\sum_{i \in A^{\backslash}} \sum_{j=1}^{p} x_{ij}}, i \in A \end{cases} \tag{4}$$

and the point estimator of $\bar{y}_{mR}$ becomes

$$\bar{y}_{mR} = \bar{y}_r \prod_{j=1}^{p} \left(\frac{\bar{x}_{nj}}{\bar{x}_{rj}} \right) \tag{5}$$

Which is clearly multi-ratio type estimator as proposed by Olkin (1958).

The estimator obtained from the multi-ratio method of imputation has shown to remain better than the estimator obtained from the multi-mean method of imputation.

2.3. Multi-Power Transformation

Singh and Deo (2003) suggested this method of imputation where,

$$y_i^{\star} = \begin{cases} y_i, i \in A \\ \bar{y}_r \left[n \prod_{j=1}^{p} \left(\frac{\bar{x}_{nj}}{\bar{x}_{rj}} \right)^{\alpha_j} - r \right] \frac{\sum_{j=1}^{p} x_{ij}}{\sum_{i \in A^{\backslash}} \sum_{j=1}^{p} x_{ij}}, i \in A^{\backslash} \end{cases} \tag{6}$$

whereα_j is a suitably chosen constant, such that the variance of the resultant estimator is minimum, and $\prod_{j=1}^{p} x_j = x_1 x_2 \ldots x_p$denote the product of p-terms. Under this method of imputation, the point estimator of$\bar{y}_s$ becomes

$$\bar{y}_{mP} = \bar{y}_r \prod_{j=1}^{p} \left(\frac{\bar{x}_{nj}}{\bar{x}_{rj}} \right)^{\alpha_j} \tag{7}$$

Which is a generalization of Srivastava (1967) estimator for multi-auxiliary information. In these situations, we are suggesting an estimator as

$$\alpha_j = \frac{\bar{x}_{rj} s_{x_j y}}{\bar{y}_r s_{x_j}^2}$$

Since $S_{x_j}^2$ *is a variance* of Auxiliary variable x_j and S_{x_jy} is a covariance of Multi-Auxiliary variables following Cochran (1977), the minimum variance of the estimator $\overline{y}_{mP}$ is given by

$$V(\overline{y}_{mP}) = \left(\frac{1}{n} - \frac{1}{N}\right) S_y^2 \left(1 - R_{y.x_1x_2\dots x_p}^2\right) \quad (8)$$

where $R_{y.x_1x_2\dots x_p}$ denotes the multiple correlation coefficient.

Following Rao and Sitter(1995), it is not clear how to use multi-auxiliary information while doing imputation with ratio method of imputation. Use of multi-auxiliary information in survey sampling has more practical use than using the single variable.

Almongy(2012), proved theoretically that the estimator obtained from the power transformation method of imputation has shown to remain better than the estimator obtained from the ratio method of imputation and hence the mean method of imputation. One can easily observe that if $\alpha_j = 1 \; \forall \; j = 1,2,\dots,p$ then, the multi-power method of imputation becomes the multi-ratio estimator. The multivariate product estimator can be easily derived by choosing $\alpha_j = -1 \; \forall \; j = 1,2,\dots,p$.

3. EMPIRICAL STUDY

For the purpose of the empirical study, we consider two types of population finite populations (Real Data), and infinite populations (Simulation). The method discussed in the previous section is not practicable if the optimum value of α_j is unknown, but fortunately the optimum value of α_j is given.

3.1. Real Data

Case 1: This study will show that the multi-power transformation method over the multi-ratio method of imputation and hence the multi-mean method of imputation, we resort to the empirical study with finite populations available. We consider a finite population of N = 15units given by Neter. et al (1983). We select all possible samples of n=7, 6, 5 units, which results in

$M = \binom{15}{n_i} = 6435, 5005, 3003\ samples\ ;\ i = 1,2,3$ respectively and we remove $m_i = 1, 2, 3, 4$ units randomly from each sample corresponding to the study variable y. Then the removed units were imputed with three methods:

- Multi-Mean method, $\overline{y}_0$(say).
- Multi-Ratio (or product) method, $\overline{y}_R$, depending upon the sign of correlation.
- Multi-power transformation method with $\alpha = \hat{\alpha}$, say $\overline{y}_P$.

$$RE.j = \frac{\sum_{s=1}^{M}\left[(\overline{y}_0)_s - \overline{Y}\right]^2}{\sum_{s=1}^{M}\left[(\overline{y}_j)_s - \overline{Y}\right]^2} \times 100, j = R, P \quad (9)$$

The relative efficiency of the multi-ratio ($RE.R$) and the multi-power ($RE.P$) with respect to multi-mean method of imputation is shown in Table 1. The same process is repeated with other finite populations (Table 2.) as shown in Table 1.

Table 1: Relative Efficiency of the Multi-Ratio and Multi-Power Methods of Imputation with Respect Tomulti-Mean Method of Imputation

	$m_1 = 1$		$m_2 = 2$		$m_3 = 3$		$m_4 = 4$	
	RE.R	RE.P	RE.R	RE.P	RE.R	RE.P	RE.R	RE.P
n=7	119.168	121.874	121.851	128.064	236.372	256.289	374.497	415.251
n=6	122.540	124.013	136.771	143.036	272.531	280.616	472.615	548.634
n=5	112.079	114.93	206.593	220.619	273.388	364.9043	531.497	652.465

Case 2: In this section, we consider a finite population of $N = 25$ units given by Montgomery and et al (2010). We select all possible samples of $n = 6, 5$ units, which results in

$M = \binom{25}{n_i} = 177100, 53130$ samples ; i = 1,2 respectively and we remove$m_i = 1, 2, 3$ units randomly from each sample corresponding to the study variable y.

Table 2: Relative Efficiency of the Multi-Ratio and Multi-Power Methods of Imputation with Respect to Multi-Mean Method of Imputation

	$m_1 = 1$		$m_2 = 2$		$m_3 = 3$	
	RE.R	RE.P	RE.R	RE.P	RE.R	RE.P
n=6	100.226	109.688	208.889	479.416	241.482	501.321
n=5	100.492	115.697	251.018	512.996	243.172	518.509

We notice the greater the number of missing then increase of efficiency obviously, since if decrease value of a number of sample size and increase value of the number of missing data then increase efficiency. Almongy (2012), concluded that there are no significant differences between the relative efficiency of the estimation methods, which are presented in this paper when we find that the number of missing units is very few.

3.2. Simulation

The size N of these populations is unknown. We generated n random numbers, $y_i^\star$, $i = 1,2\ldots,n$, from a transformed variables, given by

$$y_i = 10.0 + \sqrt{S_y^2(1-\rho_{x_1,y}{}^2)}y_i^\star + \rho_{x_1,y}S_y x_{i1}^\star + \sqrt{S_y^2(1-\rho_{x_2,y}{}^2)}y_i^\star + \rho_{x_2,y}S_y x_{i2}^\star \quad (10)$$

and$x_{i1} = 50.0 + S_{x_1}x_{i1}^\star$ and $x_{i2} = 50.0 + S_{x_2}x_{i2}^\star$ for different values of the correlation coefficient $\rho_{x_1,y}$ and $\rho_{x_2,y}$and $\bar{Y} = 10.0$. We generate 10,000 samples each of size n. From theKth sample of n units, we removed randomly $(n - r)$ units and the remaining sample units were considered to be responding. The missing values are imputed by using different methods of imputation. The empirical mean square error of the resultant estimators is computed as

$$MSE\left(\bar{y}_j\right) = \frac{1}{10{,}000}\sum_{k=1}^{10{,}000}\left[\bar{y}_{j_k} - \bar{Y}\right]^2, j = m, R, P \quad \textbf{(11)}$$

The relative efficiency of the estimators based on proposed methodwith respect to usual estimator is calculated as

$$RE.j = \frac{MSE(\bar{y}_m)}{MSE(\bar{y}_j)} \times 100, j = R, P \quad \textbf{(12)}$$

The results obtained are shown in Table 3. We conclude that the estimator $\bar{y}_P$ remains better than $\bar{y}_j, j = m, R$. Due to symmetric relationship of the efficiency of the multi-ratio (RE.R) and the multi-power (RE.P) of estimator with respect to sample mean, A the nonresponse rate Pr.m is (25%, 40%, 50%, 60% and 75%)from all samples, the relative efficiency figures remains almost the same for the given value of correlation coefficient. For example in Table 3, let that the $y, x_1 and x_2$ have gamma distribution with a parameters y and $x_1 \sim gamma(2, 18)$ and $x_2 \sim gamma(2,10)$, as shown in the following tables.

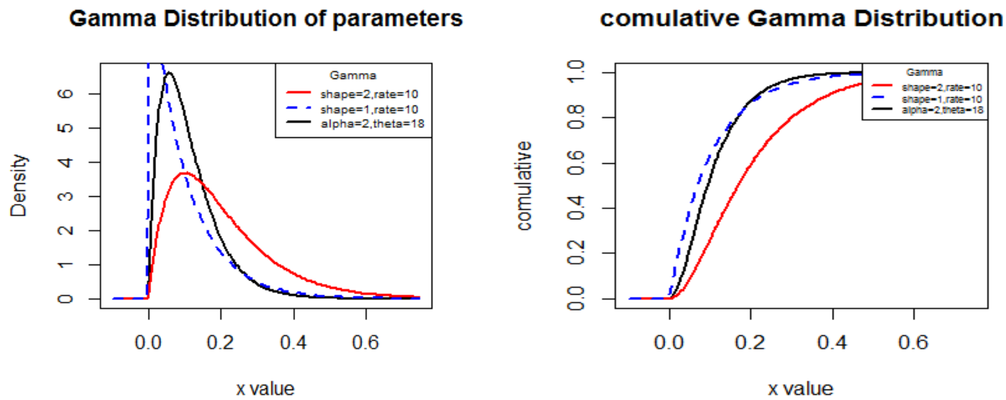

Figure 1: Plot Gamma Distribution

From table, 3.we find that, if Pr.m=25% rate of non-response is available then the gain in efficiency of the multi-ratio estimator remains between 13% to 17% and the multi-power estimator remains between 15% to 26% for $\rho_{yx_1} = 0.5$ and $\rho_{yx_2} = 0.5$. As the value of correlations coefficient increases to 0.9, then the corresponding values of the gain in efficiency of the multi-ratio estimator lies between 147% to 156%, but that of the multi-power estimator lies between 198% to 212%.

From table, 3.We find that, if $\text{Pr.m} = 40\%$ rate of non-response is available then the gain in efficiency of the multi-ratio estimator remains between 13% to 18% and the multi-power estimator remains between 15% to 27% for $\rho_{yx_1} = 0.5$ and $\rho_{yx_2} = 0.5$. As the value of correlations coefficient increases to 0.9, then the corresponding values of the gain in efficiency of the multi-ratio estimator lie between 145% to 156%, but that of the multi-power estimator lie between 196% to 222%, and so on.

Table 3: Relative Efficiency of Multi-Ratio and Multi-Power Method of Multi-Auxiliary Variables

$\rho_{yx_1} = 0.5, \rho_{yx_2} = 0.5$, y,and x_1 Have Gamma~(2,18),x_2 Has Gamma~$(2,10)$										
	Pr.m=25%		Pr.m=40%		Pr.m=50%		Pr.m=60%		Pr.m=75%	
	RE.R	RE.P	RE.R	RE.P	RE.R	RE.P	RE.R	RE.P	RE.R	RE.P
n=20	113.94	126.256	113.172	127.102	110.906	124.2	113.318	126.641	111.859	123.541
n=50	114.826	120.292	116.485	122.082	115.255	120.322	115.994	120.762	113.425	118.925
n=80	118.482	121.172	116.988	120.36	114.448	118.03	111.753	116.161	112.787	116.863
n=100	116.92	119.922	118.378	121.002	115.851	119.377	114.769	118.418	114.077	118.134
n=150	114.813	118.175	116.359	118.533	115.681	118.218	115.503	118.731	114.376	117.144
n=200	116.938	117.8	116.06	117.819	117.059	119.458	116.897	119.019	116.348	118.117
n=250	113.531	116.54	113.28	115.616	114.786	116.841	115.016	117.185	115.003	116.857
n=300	114.309	115.64	115.116	116.861	115.402	116.948	114.042	115.769	115.729	117.306
n=350	115.321	117.066	115.345	117.099	115.909	117.013	114.681	115.902	116.821	117.673
n=400	115.256	116.526	116.346	117.082	115.273	117.28	115.338	117.347	114.689	117.721
$\rho_{yx_1} = 0.6, \rho_{yx_2} = 0.6$, y,and x_1 Have Gamma~(2,18),x_2 Has Gamma~$(2,10)$										
	Pr.m=25%		Pr.m=40%		Pr.m=50%		Pr.m=60%		Pr.m=75%	
	RE.R	RE.P	RE.R	RE.P	RE.R	RE.P	RE.R	RE.P	RE.R	RE.P
n=20	124.915	138.702	124.412	138.217	122.367	136.652	122.142	136.611	124.725	139.062
n=50	126.407	131.799	126.851	130.965	127.926	132.409	127.05	131.921	126.459	130.901
n=80	129.987	133.175	128.83	131.101	127.5	129.393	127.055	129.056	125.604	128.366
n=100	126.782	128.68	128.33	131.639	130.032	133.288	128.258	131.343	127.339	129.853

n=150	129.857	131.076	130.979	131.759	128.278	129.67	128.497	130.09	131.175	132.094
n=200	126.279	127.498	125.353	126.553	127.235	128.741	126.038	127.497	124.365	125.669
n=250	127.74	128.554	128.287	129.136	128.17	128.903	126.224	126.806	127.401	127.984
n=300	131.903	133.219	132.579	133.425	132.63	133.394	131.352	132.245	127.385	128.163
n=350	126.346	126.749	125.251	125.727	127.764	128.565	127.197	128.197	127.384	128.667
n=400	129.106	129.795	128.012	128.358	124.131	124.71	123.557	123.775	125.34	126.001

Follow Table 3

$\rho_{yx_1} = 0.7, \rho_{yx_2} = 0.7$, y,and x_1 Have Gamma~(2,18),x_2 Has Gamma~$(2, 10)$										
	Pr.m=25%		Pr.m=40%		Pr.m=50%		Pr.m=60%		Pr.m=75%	
	RE.R	RE.P	RE.R	RE.P	RE.R	RE.P	RE.R	RE.P	RE.R	RE.P
n=20	144.691	158.494	143.481	161.285	141.819	156.864	144.241	159.365	140.184	153.059
n=50	144.237	151.053	143.722	150.942	144.134	150.479	145.524	152.497	146.532	154.638
n=80	147.986	153.613	147.791	153.173	144.829	149.409	144.379	148.703	145.219	148.845
n=100	148.696	153.081	145.736	150.067	144.176	148.773	142.709	146.496	142.213	144.7
n=150	145.992	148.548	147.264	150.104	146.465	148.578	146.917	149.261	147.075	150.224
n=200	142.893	144.209	144.857	146.533	143.645	145.745	142.223	143.509	142.3	143.577
n=250	146.481	149.057	144.669	147.312	146.106	149.072	147.053	149.995	148.776	152.237
n=300	147.941	150.484	149.39	152.95	151.593	155.6	149.839	153.168	148.345	150.823
n=350	146.895	148.669	143.7	145.853	141.975	143.705	143.382	145.146	142.281	143.421
n=400	151.903	154.524	149.569	152.02	150.312	153.401	149.705	152.562	146.419	148.806
$\rho_{yx_1} = 0.9, \rho_{yx_2} = 0.9$, y,and x_1 Have Gamma~(2,18),x_2 Has Gamma~$(2, 10)$										
	Pr.m=25%		Pr.m=40%		Pr.m=50%		Pr.m=60%		Pr.m=75%	
	RE.R	RE.P	RE.R	RE.P	RE.R	RE.P	RE.R	RE.P	RE.R	RE.P
n=20	255.647	309.746	251.889	321.866	251.577	314.671	247.474	302.013	239.466	294.802
n=50	248.205	299.51	255.597	316.21	252.363	307.521	251.181	309.801	247.498	297.737
n=80	248.458	304.11	245.272	296.609	249.226	303.727	250.659	306.127	250.73	303.693
n=100	253.441	314.397	254.62	315.017	258.738	316.016	260.46	319.782	256.715	308.865
n=150	257.933	325.447	251.88	314.917	252.704	319.755	254.056	324.094	252.739	311.276
n=200	250.349	313.303	253.923	318.09	255.437	325.108	249.156	311.404	254.364	311.679
n=250	254.639	320.805	251.657	305.747	250.112	306.831	249.945	306.72	251.635	313.634
n=300	253.604	312.218	253.149	317.525	257.109	320.516	252.256	310.502	255.717	313.455
n=350	256.906	323.066	256.677	319.67	249.038	309.514	250.856	315.726	251.403	311.992
n=400	250.77	310.425	251.083	311.119	251.674	309.8	247.688	303.794	245.643	298.136

Follow Table 3

$\rho_{yx_1} = 0.7, \rho_{yx_2} = 0.9$, y,and x_1 Have Gamma~(2,18),x_2 Has Gamma~$(2, 10)$										
	Pr.m=25%		Pr.m=40%		Pr.m=50%		Pr.m=60%		Pr.m=75%	
	RE.R	RE.P	RE.R	RE.P	RE.R	RE.P	RE.R	RE.P	RE.R	RE.P
n=20	178.429	204.721	178.529	197.848	178.29	199.954	182.389	204.035	175.957	191.411
n=50	187.301	206.875	184.535	202.402	184.972	200.603	181.339	196.7	179.673	194.96
n=80	181.536	199.476	182.394	198.999	180.816	198.348	184.443	200.613	185.559	203.88
n=100	187.234	204.753	183.377	200.24	185.686	204.777	185.39	202.813	185.235	201.521
n=150	182.9	197.136	185.096	198.904	183.64	197.456	186.37	202.046	184.031	201.304
n=200	185.03	201.392	189.952	206.616	184.365	199.583	183.802	197.689	185.156	197.878
n=250	184.821	200.985	185.948	202.046	183.866	199.473	184.686	200.005	183.083	198.55
n=300	186.437	204.479	184.703	201.648	188.195	204.328	189.824	207.222	189.77	205.112
n=350	182.175	197.858	181.346	194.346	181.585	197.181	183.757	199.245	180.083	191.997
n=400	184.243	199.042	181.55	196.284	179.913	193.906	181.452	194.423	182.822	197.902

Table 4: Relative Efficiency of Multi-Ratio and Multi-Power Method of Multi-Auxiliary Variables

$\rho_{yx_1} = 0.5, \rho_{yx_2} = 0.5, y$,and x_1 Have Gamma~(2,18),x_2 hasexp~(10)										
	Pr.m=25%		Pr.m=40%		Pr.m=50%		Pr.m=60%		Pr.m=75%	
	RE.R	RE.P	RE.R	RE.P	RE.R	RE.P	RE.R	RE.P	RE.R	RE.P
n=20	145.602	157.956	145.069	158.472	144.465	158.157	142.689	156.531	137.673	148.186
n=50	148.063	154.185	146.254	151.857	147.805	153.889	146.328	151.118	145.496	150.911
n=80	149.217	154.891	146.754	151.623	146.66	151.869	146.848	152.692	146.731	153.673
n=100	143.346	146.596	146.997	149.411	145.738	148.822	144.338	147.412	144.817	148.011
n=150	146.349	150.477	148.282	151.988	147.542	151.218	145.926	148.867	144.159	146.831
n=200	148.56	152.318	149.279	152.905	147.356	150.645	146.866	149.486	146.74	149.198
n=250	146.816	150.307	145.224	147.924	146.31	149.033	147.037	149.785	149.19	151.864
n=300	148.334	151.344	148.747	152.118	146.775	149.375	145.886	148.191	144.314	145.949
n=350	148.521	151.218	146.555	148.904	147.087	148.884	145.925	147.174	143.826	144.83
n=400	147.995	149.954	143.825	144.825	147.054	148.827	146.63	148.428	150.683	153.381
Follow Table 4.										
$\rho_{yx_1} = 0.9, \rho_{yx_2} = 0.9, y$,and x_1 Have Gamma~(2,18),x_2 hasexp~(10)										
	Pr.m=25%		Pr.m=40%		Pr.m=50%		Pr.m=60%		Pr.m=75%	
	RE.R	RE.P	RE.R	RE.P	RE.R	RE.P	RE.R	RE.P	RE.R	RE.P
n=20	258.368	315.322	257.255	321.258	253.352	308.809	244.833	289.963	243.773	288.824
n=50	260.073	323.005	254.555	308.073	255.633	305.477	255.734	308.704	251.716	301.142
n=80	252.139	308.61	251.156	310.376	256.656	322.752	256.192	314.355	248.415	297.119
n=100	252.568	314.421	250.479	309.809	252.724	312.775	251.299	309.121	248.772	304.829
n=150	257.726	328.057	260.97	327.815	258.67	324.41	254.96	321.124	258.133	328.071
n=200	252.426	311.992	253.713	310.849	256.345	316.063	260.469	328.342	259.068	323.096
n=250	250.313	309.643	253.647	318.855	249.75	310.119	248.994	307.026	252.788	313.212
n=300	251.298	312.276	250.931	308.007	252.549	312.682	259.289	323	252.938	314.576
n=350	249.946	302.919	252.802	308.567	250.674	309.913	251.518	313.752	251.706	312.489
n=400	252.075	319.905	256.293	327.906	255.399	319.438	251.348	308.671	259.17	317.669
$\rho_{yx_1} = 0.7, \rho_{yx_2} = 0.9, y$,and x_1 Have Gamma~(2,18),x_2 Has Exp~(10)										
	Pr.m=25%		Pr.m=40%		Pr.m=50%		Pr.m=60%		Pr.m=75%	
	RE.R	RE.P	RE.R	RE.P	RE.R	RE.P	RE.R	RE.P	RE.R	RE.P
n=20	184.198	211.746	182.875	206.854	184.273	207.394	179.321	202.271	174.777	194.697
n=50	181.48	196.863	182.216	198.841	181.976	197.692	178.326	192.859	180.239	194.383
n=80	186.978	205.538	181.419	195.748	182.182	197.15	182.24	197.646	180.878	196.884
n=100	182.885	198.076	181.979	197.381	179.237	194.332	180.945	195.297	180.372	193.565
n=150	183.28	199.388	183.118	198.984	186.703	204.919	183.054	197.538	180.693	195.026
n=200	186.354	202.623	184.937	202.107	182.421	196.287	182.344	195.797	184.005	199.276
n=250	187.978	202.491	187.127	203.055	186.851	200.6	185.923	200.268	183.438	195.766
n=300	183.595	198.386	184.644	202.756	186.158	200.754	180.903	193.002	182.226	194.058
n=350	185.28	201.423	182.659	197.533	182.895	196.716	180.393	193.736	182.115	196.458
n=400	182.777	196.264	183.419	197.337	185.325	199.257	184.836	197.959	182.877	196.519

CONCLUSIONS

In most applied cases, the estimator obtained from the multi-power transformation method of imputation has shown to remain better than the estimator obtained from the multi-ratio method of imputation and hence the multi-mean method of imputation. If the non-response rate increases from 10% to 40%, then the efficiency of the method increases, where the non-response rate more than 40%, the estimated data is the original of the data, then, the efficiency of the method decreases, and if the correlation increases between the variable y and multi-auxiliary variables, then the efficiency of the method increases. That is, there are no significant differences between the relative efficiency of the estimation methods that are presented in this paper when we find that the number of missing units is very few

REFERENCES

1. *Almongy, H. (2012). Advances in some statistical methods to increase the efficiency of post-enumeration survey. PhD. Degree, Applied Statistics and Insurance Dep. Faculty of Commerce-Mansoura University.*
2. *Abdel-Aziz, A. (2015). Treating Missing Data. PhD. Degree, Mathematical Statistics Dep. Institute of Statistical Studies and ResearchCairo University.*
3. *Bratley, P., Fox, B. L., & Schrage, L. E. (2011). A guide to simulation. Springer Science & Business Media.*
4. *Cochran, W. G., & William, G. (1977). Sampling techniques. New York: John Wiley& Sons.*
5. *Garcia MR, Cebrian AA (1996) Repeated substitution method: The ratio estimator for the population variance. Metrika, 43,101-105.*
6. *Montgomery, D. C., &Runger, G. C. (2010). Applied statistics and probability for engineers. John Wiley & Sons.*
7. *Neter, J., Kutner, M. H., Nachtsheim, C. J., & Wasserman, W. (1996). Applied linear statistical models (Vol. 4, p. 318). Chicago: Irwin.*
8. *Olkin, I. (1958). Multivariate ratio estimation for finite populations. Biometrika, 45(1/2), 154-165.*
9. *Rao, J. N., & Sitter, R. R. (1995). Variance estimation under two-phase sampling with application to imputation for missing data. Biometrika, 82(2), 453-460.*
10. *Singh, S., & Horn, S. (2000). Compromised imputation in survey sampling. Metrika, 51(3), 267-276.*
11. *Singh, S. (2000). Estimation of variance of regression estimator in two phase sampling. Calcutta Statistical Association Bulletin, 50(1-2), 49-64.*
12. *Mehta, Nitu, And Vl Mandowara. "Advanced Estimator In Stratified Ranked Set Sampling Using Auxiliary Information."*
13. *Singh, S. (2001). Generalized calibration approach for estimating variance in survey sampling. Annals of the Institute of Statistical Mathematics, 53(2), 404-417.*
14. *Singh, S., &Deo, B. (2003). Imputation by power transformation. Statistical Papers, 44(4), 555-579.*
15. *Srivastava, S. K. (1967). An estimator using auxiliary information in sample surveys. Calcutta Statistical Association Bulletin, 16(2-3), 121-132.*